Practice Test - Chapter 4

Complete parts a–c for each quadratic function.
a. Find the *y*-intercept, the equation of the axis of symmetry, and the *x*-coordinate of the vertex.
b. Make a table of values that includes the vertex.
c. Use this information to graph the function.

1. $f(x) = x^2 + 4x - 7$

SOLUTION:

a. Compare the function $f(x) = x^2 + 4x - 7$ with the standard form of the quadratic function.
Here, *a* = 1, *b* = 4 and *c* = –7.
The *y*-intercept is –7.
The equation of the axis of symmetry is

$x = -\frac{b}{2a} = -\frac{4}{2(1)} = -2$.

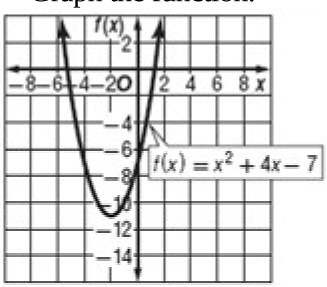
Therefore, the axis of symmetry is *x* = –2.

The *x*-coordinate of the vertex is $-\frac{b}{2a} = -2$.

b. Substitute –4, –3, –2, –1 and 0 for *x* and make the table.

x	f(x)
–4	–7
–3	–10
–2	–11
–1	–10
0	–7

c. Graph the function.



2. $f(x) = -2x^2 + 5x$

SOLUTION:

a. Compare the function $f(x) = -2x^2 + 5x$ with the standard form of the quadratic function.
Here, *a* = –2, *b* = 5 and *c* = 0.
The *y*-intercept is 0.
The equation of the axis of symmetry is

$x = -\frac{b}{2a} = -\frac{5}{2(-2)} = \frac{5}{4}$.

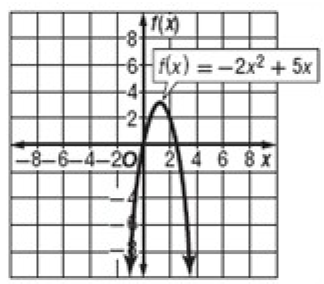
Therefore, the axis of symmetry is *x* = $\frac{5}{4}$.

The *x*-coordinate of the vertex is $-\frac{b}{2a} = \frac{5}{4}$.

b. Substitute $0, 1, \frac{5}{4}, 2$ and 3 for *x* and make the table.

x	f(x)
0	0
1	3
$\frac{5}{4}$	$\frac{25}{8}$
2	2
3	–3

c. Graph the function.



3. $f(x) = -x^2 - 6x - 9$

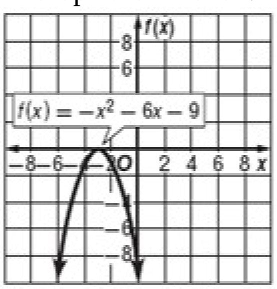
SOLUTION:

a. Compare the function $f(x) = -x^2 - 6x - 9$ with the standard form of the quadratic function.
Here, $a = -1$, $b = -6$ and $c = -9$.
The y-intercept is –9.
The equation of the axis of symmetry is
$x = -\frac{b}{2a} = -\frac{-6}{2(-1)} = -3$.
Therefore, the axis of symmetry is $x = -3$.
The x-coordinate of the vertex is $-\frac{b}{2a} = -3$.

b. Substitute –5, –4, –3, –2 and –1 for x and make the table.

x	f(x)
–5	–4
–4	–1
–3	0
–2	–1
–1	–4

c. Graph the function.



Determine whether each function has a maximum or minimum value. State the maximum or minimum value of each function.

4. $f(x) = x^2 + 10x + 25$

SOLUTION:

Compare the function $f(x) = x^2 + 10x + 25$ with the standard form of the quadratic function.
Here, $a = 1$, $b = 10$ and $c = 25$.
For this function, $a > 0$, so the graph opens up and the function has a minimum value.
The x-coordinate of the vertex is
$-\frac{b}{2a} = -\frac{10}{2(1)} = -5$.
Substitute –5 for x in the function to find the y-coordinate of the vertex.

$$\begin{aligned} f(-5) &= (-5)^2 + 10(-5) + 25 \\ &= 25 - 50 + 25 \\ &= 0 \end{aligned}$$

Therefore, the minimum value of the function is 0.

5. $f(x) = -x^2 + 6x$

SOLUTION:

Compare the function $f(x) = -x^2 + 6x$ with the standard form of the quadratic function.
Here, $a = -1$, $b = 6$ and $c = 0$.
For this function, $a = -1$, so the graph opens down and the function has a maximum value.
The x-coordinate of the vertex is
$-\frac{b}{2a} = -\frac{6}{2(-1)} = 3$.
Substitute 3 for x in the function to find the y-coordinate of the vertex.

$$\begin{aligned} f(3) &= -(3)^2 + 6(3) \\ &= -9 + 18 \\ &= 9 \end{aligned}$$

Therefore, the maximum value of the function is 9.

Solve each equation using the method of your choice. Find exact solutions.

6. $x^2 - 8x - 9 = 0$

SOLUTION:

$$x^2 - 8x - 9 = 0$$
$$(x-9)(x+1) = 0$$
$$x+1=0 \text{ or } x-9=0$$
$$x=-1 \quad \text{or} \quad x=9$$

7. $-4.8x^2 + 1.6x + 24 = 0$

SOLUTION:

$$x = \frac{-1.6 \pm \sqrt{(1.6)^2 - 4(-4.8)(24)}}{2(-4.8)}$$
$$= \frac{-1.6 \pm \sqrt{2.56 + 460.8}}{-9.6}$$
$$= \frac{-1.6 \pm \sqrt{463.36}}{-9.6}$$
$$= \frac{-1.6 \pm \sqrt{2.56(181)}}{-9.6}$$
$$= \frac{-1.6\left(1 \pm \sqrt{181}\right)}{-9.6}$$
$$= \frac{1 \pm \sqrt{181}}{6}$$

8. $12x^2 + 15x - 4 = 0$

SOLUTION:

$$x = \frac{-15 \pm \sqrt{15^2 - 4(12)(-4)}}{2(12)}$$
$$= \frac{-15 \pm \sqrt{225 + 192}}{24}$$
$$= \frac{-15 \pm \sqrt{417}}{24}$$

9. $x^2 - 7x - \frac{17}{4} = 0$

SOLUTION:

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)\left(-\frac{17}{4}\right)}}{2(1)}$$
$$= \frac{7 \pm \sqrt{49 + 17}}{2}$$
$$= \frac{7 \pm \sqrt{66}}{2}$$

10. $4x^2 + x = 3$

SOLUTION:

$$4x^2 + x = 3$$
$$4x^2 + x - 3 = 3 - 3$$
$$4x^2 + x - 3 = 0$$
$$x = \frac{-(1) \pm \sqrt{(1)^2 - 4(4)(-3)}}{2(4)}$$
$$= \frac{-1 \pm \sqrt{1 + 48}}{8}$$
$$= \frac{-1 \pm \sqrt{49}}{8}$$
$$= \frac{-1 \pm 7}{8}$$
$$x = -1, \frac{3}{4}$$

11. $-9x^2 + 40x + 84 = 0$

SOLUTION:

$$x = \frac{-40 \pm \sqrt{40^2 - 4(-9)(84)}}{2(-9)}$$
$$= \frac{-40 \pm \sqrt{1600 + 3024}}{-18}$$
$$= \frac{-40 \pm 68}{-18}$$
$$x = 6, -\frac{14}{9}$$

12. **PHYSICAL SCIENCE** Parker throws a ball off the top of a building. The building is 350 feet high and the initial velocity of the ball is 30 feet per second. Find out how long it will take the ball to hit the ground by solving the equation
$-16t^2 - 30t + 350 = 0$.

SOLUTION:

Substitute 0 for $h(t)$ and find the roots.

$$-16t^2 - 30t + 350 = 0$$
$$t = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(-16)(350)}}{2(-16)}$$
$$= \frac{30 \pm \sqrt{23300}}{-32}$$
$$t = 3.8 \text{ or } t = -5.71$$

The value of t should be positive. Therefore, the ball will reach the ground in about 3.83 s.

13. **MULTIPLE CHOICE** Which equation below has roots at –6 and $\frac{1}{5}$?

A $0 = 5x^2 - 29x - 6$

B $0 = 5x^2 + 31x + 6$

C $0 = 5x^2 + 29x - 6$

D $0 = 5x^2 - 31x + 6$

SOLUTION:

Roots of the equation $0 = 5x^2 - 29x - 6$ are $-\frac{1}{5}$ and 6.

Roots of the equation $0 = 5x^2 + 31x + 6$ are $-\frac{1}{5}$ and -6.

Roots of the equation $0 = 5x^2 + 29x - 6$ are $\frac{1}{5}$ and -6.

Roots of the equation $0 = 5x^2 - 31x + 6$ are $\frac{1}{5}$ and 6.

Therefore, option C is the correct answer.

14. **PHYSICS** A ball is thrown into the air vertically with a velocity of 112 feet per second. The ball was released 6 feet above the ground. The height above the ground t seconds after release is modeled by $h(t) = -16t^2 + 112t + 6$.

a. When will the ball reach 130 feet?

b. Will the ball ever reach 250 feet? Explain.

c. In how many seconds after its release will the ball hit the ground?

SOLUTION:

a. Substitute 130 for $h(t)$ and find the roots of the equation.

$$-16t^2 + 112t + 6 = 130$$
$$-16t^2 + 112t - 124 = 0$$
$$t = \frac{-112 \pm \sqrt{112^2 - 4(-16)(-124)}}{2(-16)}$$
$$= \frac{-112 \pm \sqrt{4608}}{-32}$$
$$t = 5.6 \text{ or } t = 1.4$$

The ball will reach 130 feet at about 1.4 s and 5.6 s.

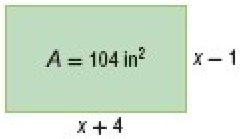
b. No; if you graph the function, the vertex in 202 units above the horizontal axis. So, the height will never be 250.

c. Substitute 0 for $h(t)$ and find the roots.

$$-16t^2 + 112t + 6 = 0$$
$$t = \frac{-112 \pm \sqrt{112^2 - 4(-16)(6)}}{2(-16)}$$
$$= \frac{-112 \pm \sqrt{12928}}{-32}$$
$$t \approx 7 \text{ or } t = -0.05$$

The value of t should be positive. Therefore, the ball will reach the ground in about 7 s.

15. The rectangle below has an area of 104 square inches. Find the value of x and the dimensions of the rectangle.



SOLUTION:

Area of the given rectangle is $(x+4)(x-1)$.

$$(x-1)(x+4)=104$$
$$x^2+3x-4=104$$
$$x^2+3x-108=0$$
$$(x+12)(x-9)=0$$

Therefore, $x = -12$ or $x = 9$.
The value of x should positive.
Therefore, $x = 9$.
The dimensions of the rectangle are 8 inches by 13 inches.

Simplify.

16. $(3-4i)-(9-5i)$

SOLUTION:

$$\begin{aligned}(3-4i)-(9-5i)&=3-4i-9+5i\\&=-6+i\end{aligned}$$

17. $\dfrac{4i}{4-i}$

SOLUTION:

$$\begin{aligned}\frac{4i}{4-i}&=\frac{4i}{4-i}\cdot\frac{4+i}{4+i}\\&=\frac{16i+4i^2}{4^2-i^2}\\&=\frac{16i+4(-1)}{16-(-1)}\\&=\frac{-4+16i}{17}\\&=-\frac{4}{17}+\frac{16}{17}\end{aligned}$$

18. **MULTIPLE CHOICE** Which value of c makes the trinomial $x^2 - 12x + c$ a perfect square trinomial?

F 6
G 12
H 36
J 144

SOLUTION:

To make the given trinomial as a perfect square, add the square of half of the coefficient of x.

Square of half of the coefficient of x is $\left(\frac{-12}{2}\right)^2=36$

Therefore, option H is the correct answer.

Complete parts a–c for each quadratic equation.
a. Find the value of the discriminant.
b. Describe the number and type of roots.
c. Find the exact solution by using the Quadratic Formula.

19. $6x^2+7x=0$

SOLUTION:

a. Compare the equation with the standard quadratic equation.
Here, $a=6$, $b=7$ and $c=0$.

$$\begin{aligned}b^2-4ac&=7^2-4(6)(0)\\&=49\end{aligned}$$

b. The value of the discriminant is positive and a perfect square. So, the equation has two rational roots.

c.

$$6x^2+7x=0$$
$$x(6x+7)=0$$
$$6x+7=0 \quad \text{or} \quad x=0$$
$$x=-\frac{7}{6} \quad \text{or} \quad x=0$$

20. $5x^2 = -6x + 1$

SOLUTION:

a.

$$5x^2 = -6x + 1$$
$$5x^2 + 6x - 1 = 0$$

Compare the equation with the standard quadratic equation.

Here, $a = 5, b = 6 \text{ and } c = -1$.

$$b^2 - 4ac = 6^2 - 4(5)(-1)$$
$$= 36 + 20$$
$$= 56$$

b. The value of the discriminant is positive and not a perfect square. So, the equation has two irrational roots.

c.

$$x = \frac{-6 \pm \sqrt{6^2 - 4(5)(-1)}}{2(5)}$$
$$= \frac{-6 \pm \sqrt{56}}{10}$$
$$= \frac{-3 \pm \sqrt{14}}{5}$$

21. $2x^2 + 5x - 8 = -13$

SOLUTION:

a.

$$2x^2 + 5x - 8 = -13$$
$$2x^2 + 5x - 8 + 13 = 0$$
$$2x^2 + 5x + 5 = 0$$

Compare the equation with the standard quadratic equation.

Here, $a = 2, b = 5 \text{ and } c = 5$.

$$b^2 - 4ac = 5^2 - 4(2)(5)$$
$$= 25 - 40$$
$$= -15$$

b. The value of the discriminant is negative. So, the equation has two complex roots.

c.

$$x = \frac{-5 \pm \sqrt{5^2 - 4(2)(5)}}{2(5)}$$
$$= \frac{-5 \pm \sqrt{-15}}{10}$$
$$= \frac{-5 \pm i\sqrt{15}}{10}$$

Write each quadratic function in vertex form. Then identify the vertex, axis of symmetry, and direction of opening.

22. $3x^2 + 6x = 2 + y$

SOLUTION:

$$3x^2 + 6x = 2 + y$$
$$3(x^2 + 2x) = 2 + y$$
$$3(x^2 + 2x + 1) - 3 = 2 + y$$
$$3(x + 1)^2 - 3 = 2 + y$$
$$3(x + 1)^2 - 5 = y$$

The vertex of the function is (–1, 5).

The axis of the symmetry is $x = -1$.

Since the coefficient of the x^2 is positive the graph opens upwards.

23. $x^2 + 9x + \frac{81}{4} = y$

SOLUTION:

$x^2 + 9x + \frac{81}{4} = y$

$\left(x + \frac{9}{2}\right)^2 = y$

The vertex of the function is $\left(-\frac{9}{2}, 0\right)$.

The axis of the symmetry is $x = -\frac{9}{2}$.

Since the coefficient of the term x^2 is positive the graph opens upwards.

24. Graph the quadratic inequality $0 < -3x^2 + 4x + 10$.

SOLUTION:

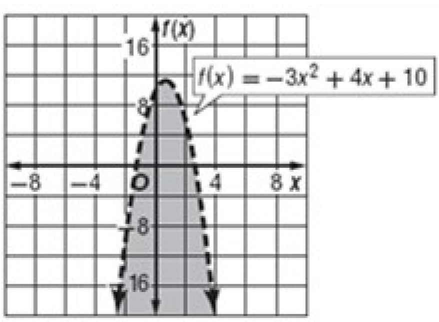
First graph the related equation. The parabola should be dashed. Next choose a test point such as (0, 0) and determine if that is a solution to the inequality.

$f(x) < -3x^2 + 4x + 10$

$0 < -3(0)^2 + 4(0) + 10$

$0 < 10$

The solution of the inequality contains (0, 0). Shade the region of the graph that contains this point.



Solve each inequality by using a graph or algebraically.

25. $x^2 + 6x > -5$

SOLUTION:

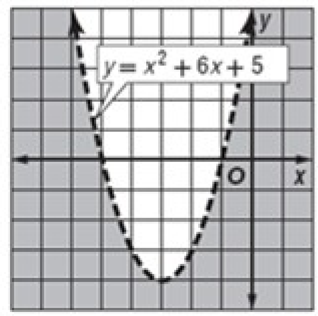
$x^2 + 6x > -5$

$x^2 + 6x + 5 > -5 + 5$

$x^2 + 6x + 5 > 0$

$0 < x^2 + 6x + 5$

Graph the inequality.



Therefore, the solution is $\{x \mid x < -5 \text{ or } x > -1\}$.

26. $4x^2 - 19x \leq -12$

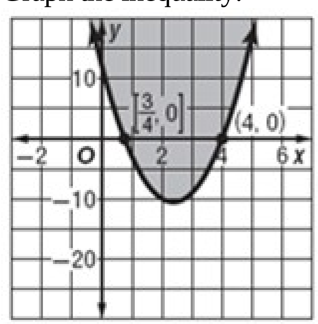
SOLUTION:

$4x^2 - 19x \leq -12$

$4x^2 - 19x + 12 \leq -12 + 12$

$4x^2 - 19x + 12 \leq 0$

Graph the inequality.



Therefore, the solution is $\left\{x \middle| \frac{3}{4} \leq x \leq 4\right\}$.